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String theory on AdS_3

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Abstract: It was shown by Brown and Henneaux that the classical theory of gravity on AdS_3 has an infinite-dimensional symmetry group forming a Virasoro algebra. More recently, Giveon, Kutasov and Seiberg (GKS) constructed the corresponding Virasoro generators in the first-quantized string theory on AdS_3. In this paper, we explore various aspects of string theory on AdS_3 and study the relation between these two works. We show how semi-classical properties of the string theory reproduce many features of the AdS/CFT duality. Furthermore, we examine how the Virasoro symmetry of Brown and Henneaux is realized in string theory, and show how it leads to the Virasoro Ward identities of the boundary CFT. The Virasoro generators of GKS emerge naturally in this analysis. Our work clarifies several aspects of the GKS construction: why the Brown-Henneaux Virasoro algebra can be realized on the first-quantized Hilbert space, to what extent the free-field approximation is valid, and why the Virasoro generators act on the string worldsheet localized near the boundary of AdS_3. On the other hand, we find that the way the central charge of the Virasoro algebra is generated is different from the mechanism proposed by GKS.

Keywords: p-branes, Black Holes in String Theory.

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Contents

1. Introduction

It was shown by Brown and Henneaux [1] that the semi-classical theory of gravitation on three-dimensional anti-de Sitter space (AdS_3) possesses an infinite-dimensional symmetry algebra of Virasoro type. The realization of this Virasoro algebra has recently been clarified in light of the AdS/CFT duality [2, 3, 4], according to which string/M-theory on a $(p+1)$-dimensional anti-de Sitter space times a compact space is equivalent to a p-dimensional conformal field theory (CFT_p). The case of AdS_3 was studied in more detail in [5, 6, 7, 8]. More recently, Giveon, Kutasov and Seiberg (GKS) found that the Brown-Henneaux Virasoro algebra is realized on the first-quantized string theory on AdS_3, shedding further light on the duality [9].

The main purpose of this paper is to clarify the relation between the Brown-Henneaux Virasoro algebra and the Virasoro generators constructed by GKS. In the construction of GKS, the Virasoro algebra acts on the first-quantized string Hilbert space. However, as shown in [5, 6, 7, 8], the Brown-Henneaux Virasoro operators are creation and annihilation operators of gravitons in AdS_3, and as such they are realized on the second-quantized Hilbert space of strings. It was not clear how to reconcile these two points of view. In addition, GKS assume that the string worldsheet is localized near the boundary of AdS_3 and winds around the boundary. It is not obvious why we need (and need only) consider such worldsheet configurations. In this paper, we will give answers to these questions, and along the way, we will recover many features of the AdS/CFT duality directly from the worldsheet theory of strings on AdS_3.

As by-products of this analysis, we gain new insights into the structure of two-dimensional sigma models with non-compact target spaces such as AdS_3. In the case of AdS_3 with the Euclidean-signature metric, the sigma model is unitary and its Hilbert space is equipped with a positive-definite inner product. Since AdS_3 has an $SO(3,1) \simeq SL(2,C)$ isometry group, the Hilbert space should decompose into a direct sum of unitary representations of $SL(2,C)$. To our surprise, we find that the AdS/CFT duality implies that vertex operators of the sigma model belong to non-unitary representations of $SL(2,C)$. This is not a contradiction, and appears to be a generic phenomenon in non-compact sigma models. It is closely related to the absence of the state-operator correspondence in the Liouville model [10], where it is known that normalizable states make up the Hilbert space, while non-normalizable states correspond to operators. In the AdS_3 case, unitary representations of $SL(2,C)$ are realized by normalizable functions on AdS_3, whereas non-normalizable functions give non-unitary representations. Thus it is reasonable, by analogy with the Liouville model, that the vertex operators of the sigma model belong to non-unitary representations.

This paper is organized as follows.

In section 2, we briefly summarize the duality between string theory on AdS_3 and conformal field theory in two dimensions.

In section 3, we discuss various aspects of the worldsheet theory of strings on Euclidean AdS_3, including the $SL(2,C)$ symmetry and the vertex operator construction. We show that the worldsheet vertex operators are closely related to bulk-boundary Green's functions in target space.

In section 4, we perform a semi-classical analysis of correlation functions of primary fields, and show that the vertex operators are subject to the wave function renormalization expected from the AdS/CFT duality and from the holographic identification of the regularizations [11]. The worldsheet stretches to the boundary of AdS_3 at the insertion points of the vertex operators, and can be viewed as a thickening of the target-space Feynman diagram involving bulk-boundary and bulk-bulk Green's functions.

In section 5, we define the Virasoro generators as the graviton vertex operators corresponding to Brown-Henneaux diffeomorphisms, and explain why these vertex operators do not decouple from the theory.

In section 6, we derive the Virasoro Ward identity of the boundary CFT and show how the Virasoro generators defined by GKS [9] arise.

In section 7, we consider the correlation function of two boundary stress-energy tensors and explain how the central charge appears. A crucial step is to consider disconnected worldsheets, *i.e.*, second-quantized string theory.

We end in section 8 with some conclusions.

2. The AdS_3/CFT_2 duality

Following [6], we start with type IIB string theory on $\mathbb{R}^4 \times \mathbb{R}^2 \times M^4$, where M^4 is a compact manifold ($M^4 = T^4$ or K_3), and consider Q_1 fundamental strings on $\mathbb{R}^2$ and Q_5 *NS* fivebranes on $\mathbb{R}^2 \times M^4$. In the near-horizon limit, the target space geometry is $AdS_3 \times S^3 \times M^4$, with a non-zero NS-NS 2-form. The curvature radius l_{AdS} of AdS_3 is equal to $\sqrt{Q_5} l_s$ where l_s is the string scale. The string coupling constant on AdS_3 is

$$g_3^2 = \frac{1}{Q_1 \sqrt{Q_5}} \,. \tag{2.1}$$

Thus we have the following hierarchy of scales:

$$l_{AdS} = \sqrt{Q_5} l_s = 4 Q_1 Q_5 l_p \,, \tag{2.2}$$

where $l_p = \frac{1}{4} g_3^2 l_s$ is the three-dimensional Planck length. When $Q_1 Q_5 \gg 1$, quantum gravity effects are weak. Moreover when $Q_5 \gg 1$, the α'-expansion of the worldsheet theory becomes reliable.

According to the AdS/CFT duality, this system is dual to some two-dimensional conformal field theory (CFT_2). In the original work of Brown and Henneaux [1], the central charge c of the CFT_2 is given in the low-energy gravity approximation by

$$c = \frac{3 l_{AdS}}{2 l_p} = 6 Q_1 Q_5 \,. \tag{2.3}$$

This is consistent with the S-dual of the brane-configuration, which is the D1-D5 system, whose field theory limit is a CFT_2 with $c = 6 Q_1 Q_5$ [12].

3. Worldsheet description of strings on AdS$_3$

3.1 Action and symmetry

In Euclidean AdS_3, the bosonic part of the worldsheet Lagrangian is

$$S_E = \frac{Q_5}{2\pi} \int d^2 z \left(\partial\phi \bar{\partial}\phi + e^{2\phi} \partial\bar{\gamma} \bar{\partial}\gamma \right) \,. \tag{3.1}$$

Here $(\phi, \gamma, \bar{\gamma})$ are the coordinates on AdS_3. The coordinate ϕ is real, while γ and $\bar{\gamma}$ are complex conjugates. The boundary of AdS_3 is located at $\phi = \infty$. In this sub-section we will summarize known facts about this action, based on the earlier works [9, 13, 14].

First of all, it is instructive to compare (3.1) with the corresponding action S_L for AdS_3 with Lorentzian signature. Lorentzian-signature AdS_3 is the group manifold of $SL(2,R)$; the action S_L is the Wess-Zumino-Witten (WZW) action for $SL(2,R)$ with level Q_5, and so possesses an affine $SL(2,R)\times SL(2,R)$ symmetry, with independent generators for the left- and right-movers.

Euclidean AdS_3 is the coset manifold $SL(2,C)/SU(2)$; the action S_E can be directly obtained from the $SL(2,C)$ WZW action $S_{wzw}(g)$ [13].[1] The $SL(2,C)$ WZW model has two independent affine $SL(2,C)$ symmetries, associated with left- and right-movers. The quotient by $SU(2)$ identifies the left and the right affine symmetries by complex conjugation. This can be seen as follows.

We may regard the coset $SL(2,C)/SU(2)$ as the space of 2×2 hermitian complex matrices h with unit determinant. To compare with the action (3.1), we parametrize an $SL(2,C)/SU(2)$ matrix h as

$$h=\begin{pmatrix} e^{-\phi}+\gamma\bar{\gamma}e^{\phi} & e^{\phi}\gamma \\ e^{\phi}\bar{\gamma} & e^{\phi} \end{pmatrix}. \tag{3.2}$$

The string action (3.1) is simply the $SL(2,C)$ WZW action $S_{wzw}(h)$, with h restricted to the form (3.2). By construction, the WZW action $S_{wzw}(g)$ is invariant for arbitrary $g\in SL(2,C)$ under the left and the right $SL(2,C)$ symmetries

$$g\to U(z)gV^{\dagger}(\bar{z})\,,\qquad U,\,V\in SL(2,C)\,. \tag{3.3}$$

However, $S_E=S_{wzw}(g=h)$ is invariant only under the diagonal action

$$h\to U(z)hU^{\dagger}(\bar{z})\,,\qquad U\in SL(2,C)\,, \tag{3.4}$$

since h is constrained to be hermitian. The matrix U is an arbitrary holomorphic function of z; consequently, by Noether's theorem, the corresponding currents J^a ($a=\pm,3$) are holomorphically conserved,

$$\bar{\partial}J^a=0\,. \tag{3.5}$$

So far we have discussed the classical symmetry of the action S_E. The currents J^a could receive quantum corrections, but we expect that the conservation law (3.5) still holds. There are two instances in which quantum effects can be perturbatively treated.

[1] This model has been studied in the past, owing to its relation to coset conformal field theories. It was shown in [14] that, when G and H are compact groups, the G/H model is equivalent to the product of the G model and the H^c/H model, where H^c is the complexification of H, when some BRST invariance is imposed on the product theory. It is interesting to note that, if we take $G=H=SU(2)$, we find that the topological $SU(2)/SU(2)$ model is equivalent to the product of the $SL(2,C)/SU(2)=AdS_3$ model and the $SU(2)=S^3$ model (with the BRST invariance). Before imposing the BRST invariance, the product model is nothing but the worldsheet theory of strings on $AdS_3\times S^3$. The $SU(2)/SU(2)$ model may be useful to study some topological aspects of the string theory in question.

1) When Q_5 is large, worldsheet quantum effects are suppressed by $1/Q_5$.

2) If the functional integral is dominated by contributions at large ϕ, we can use the action

$$S' = \frac{1}{4\pi}\int d^2z \left(\partial\phi\bar{\partial}\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - \beta\bar{\beta}e^{-2\phi/\alpha_+} - \frac{2}{\alpha_+}\phi\sqrt{g}R\right), \qquad (3.6)$$

where $\alpha_+ = \sqrt{2Q_5 - 4}$ and R is the curvature of the worldsheet.

The theory defined by the action S' can be shown to be equivalent to the original one, upon integrating out $(\beta, \bar{\beta})$, taking into account effects on the measure of the functional integral, and rescaling the scalar fields by $\phi \to \phi\alpha_+$, $\gamma \to \sqrt{2Q_5}\gamma$ [16]. For large ϕ, the interaction term $\beta\bar{\beta}e^{-2\phi/\alpha_+}$ is suppressed and the free-field approximation to the fields (β, γ) becomes reliable. The $SL(2,C)$ currents in this notation are given by

$$\begin{aligned} J^- &= \frac{1}{2}\beta \\ J^3 &= \frac{1}{2}\left(\beta\gamma - \alpha_+\partial\phi\right) \\ J^+ &= \frac{1}{2}\left(\beta\gamma^2 - 2\alpha_+\gamma\partial\phi - \alpha_+^2\partial\gamma\right). \end{aligned} \qquad (3.7)$$

Moreover, because of the coupling of ϕ to the worldsheet curvature R in (3.6), the effective string coupling constant depends on the coordinate ϕ (the linear dilaton background). For $\phi \to \infty$, the string coupling constant vanishes asymptotically. Thus the spacetime theory as well as the worldsheet theory is weakly coupled for $\phi \to \infty$ in this picture [9].[2]

3.2 Vertex operators

According to the AdS/CFT duality, correlation functions of CFT correspond to string amplitudes on AdS [3, 4]. It is therefore useful to study vertex operators of the AdS_3 string. Generally speaking, if a CFT has a global affine G symmetry, its

[2] It may appear that, in the opposite limit $\phi \to -\infty$, the effective string coupling constant diverges, and the spacetime theory is strongly coupled. This, however, is an artifact of the description in terms of the action S'. In the limit $\phi \to -\infty$, the transformation relating S to S' breaks down, because the factor $e^{2\phi}$ multiplying the kinetic term for γ in (3.1) vanishes. In fact, this transformation is an intermediate step in the T-duality transformation along the isometry generated by a constant shift of $(\gamma, \bar{\gamma})$. (It becomes T-duality if we write $\beta = \partial\tilde{\gamma}$ [17].) The T-duality transformation is subtle when there is a fixed point in the isometry. After T-duality, the dilaton diverges at the fixed point, but this is an artifact, if the original theory is well-defined at that point. This is the case for the string on AdS_3 since $\phi = -\infty$ is a regular boundary point on AdS_3 and the string coupling is constant, $g_s^{-2} = Q_1\sqrt{Q_5}$ in the original picture. In this paper, we will use S' only when we analyze the behavior of the functional integral for large ϕ.

vertex operators $V(z,\bar{z})$ take values in vector spaces representing the G symmetry. In the case of AdS_3, since the group $SL(2,C)$ is non-compact, we are led to consider infinite-dimensional representations as well as finite-dimensional ones. Teschner [15] introduced auxiliary coordinates $(x,\bar{x})$ to organize these representations. Because $SL(2,C)$ acts on the matrix h as $h \to UhU^\dagger$, it is natural to consider the combination

$$(1,-x)h\begin{pmatrix} 1 \\ -\bar{x} \end{pmatrix} = e^{\phi/\alpha_+}(\gamma - x)(\bar{\gamma} - \bar{x}) + e^{-\phi/\alpha_+}\,, \tag{3.8}$$

and to define the vertex operator V_j by[3]

$$V_j(z,\bar{z};x,\bar{x}) = \left((\gamma - x)(\bar{\gamma} - \bar{x})e^{\phi/\alpha_+} + e^{-\phi/\alpha_+}\right)^{2j}. \tag{3.9}$$

In the free-field approximation, it is straightforward to show that this vertex operator gives the correct operator product expansion with the $SL(2,C)$ currents,

$$J^a(z)V_j(w,\bar{w};x,\bar{x}) \sim \frac{1}{z-w} D^a V_j(w,\bar{w};x,\bar{x})\,, \tag{3.10}$$

where $a = 3, \pm$, and

$$D^- = \frac{\partial}{\partial x}\,, \qquad D^3 = x\frac{\partial}{\partial x} - j\,, \qquad D^+ = x^2\frac{\partial}{\partial x} - 2jx\,. \tag{3.11}$$

As we will show in section 6, in evaluating the operator product expansion of J^a with V_j, we can take ϕ to be arbitrarily large. Therefore the computation in (3.10) belongs to the case 2) discussed in section 3.1, and justifies the use of the free-field approximation.

The global $SL(2,C)$ symmetry of AdS_3 corresponds to the global conformal symmetry of the boundary CFT_2 generated by L_0 and $L_{\pm 1}$ [2, 6]. One can then relate the highest weight j of $SL(2,C)$ to the Virasoro highest weight h of the boundary CFT by

$$h = -j\,. \tag{3.12}$$

In [15], Teschner considered the case $j \in -1/2 + \sqrt{-1}\mathbb{R}$, corresponding to principal representations of $SL(2,C)$. These are unitary representations and therefore appear in the Hilbert space of the sigma model.

In this paper, we are interested in the situation when $h = -j$ is real, since h is a conformal weight of the boundary CFT_2. In this case, the $SL(2,C)$ representation is non-unitary, and the corresponding supergravity mode is non-normalizable. For

[3]The vertex operators of [9] correspond to the leading large ϕ part of the coefficients of the $x,\bar{x}$ expansion of V_j.

$h > 1/2$, because of the identity

$$\delta^{(2)}(z) = \frac{n-1}{\pi} \lim_{\epsilon \to 0} \frac{\epsilon^{2n-2}}{(\epsilon^2 + |z|^2)^n}\,, \tag{3.13}$$

the vertex operator V_j behaves as

$$V_{j=-h} \sim e^{2(h-1)\phi/\alpha_+} \delta^{(2)}(\gamma - x)\,, \tag{3.14}$$

near the boundary ($\phi \to \infty$) of AdS_3. That is, the vertex operator V_j has the same structure as the bulk-boundary Green's function used in the supergravity computation of CFT correlation functions [3, 4]. Of course, this is not a coincidence. In the semi-classical approximation, if a vertex operator V_j is expressed as a function of $(\phi, \gamma, \bar{\gamma})$, the operator product expansion (3.10) with the $SL(2,C)$ currents implies that V_j solves the supergravity wave equation

$$(\Delta + j(j+1))\, V_j = 0\,, \tag{3.15}$$

where Δ is the Laplacian on AdS_3, expressed in the coordinates $(\phi, \gamma, \bar{\gamma})$. The identification of vertex operators and bulk-boundary Green's functions motivates us to interpret $(x, \bar{x})$ as coordinates for the boundary CFT_2.

When j is real, the vertex operator V_j carries the $SL(2,C)$ weights $h = \bar{h} = -j$ and corresponds to a scalar field on AdS_3, such as a Kaluza-Klein excitation (on $S^3 \times M^4$) of the dilaton field. To construct a vertex operator with $h \neq \bar{h}$, corresponding to tensor fields on AdS_3, we must include derivatives of the fields $(\phi, \gamma, \bar{\gamma})$. Indeed, we will see in section 5 that the graviton vertex operator corresponding to the energy-momentum tensor $T(x)$ of CFT_2 is of this form.

We have found that the AdS/CFT duality implies that the vertex operators V_j belong to non-unitary representations of $SL(2,C)$, even though both the two-dimensional sigma model for Euclidean AdS_3 and the boundary CFT_2 are expected to be unitary theories, with Hilbert spaces of positive-definite inner product. Therefore there is no state-operator correspondence in the sigma model.[4] This phenomenon is well-known in the Liouville model. In the Liouville model, normalizable states make up the Hilbert space and non-normalizable states correspond to operators [10]. In the AdS_3 case, unitary representations of $SL(2,C)$ are realized by normalizable functions on AdS_3, whereas non-normalizable functions give non-unitary representations. Thus it is in fact reasonable, by the analogy with the Liouville model, that the vertex operators of the sigma model belong to non-unitary representations.

[4] A generalized version of the correspondence may hold if we suitably extend the notion of states and allow for analytic continuation of the quantum number j in (3.9) [15, 18].

4. Semi-classical analysis

In this section[5] we will analyze correlation functions of the vertex operators (3.9) semi-classically. We propose the correspondence[6]

$$\left\langle \prod_i \int d^2 z_i V_{j_i}(z_i,\bar{z}_i;x_i,\bar{x}_i) \right\rangle_{\text{worldsheet}} = \left\langle \prod_i V_{j_i}(x_i,\bar{x}_i) \right\rangle_{\text{boundary CFT}} . \tag{4.1}$$

(In this expression, factors coming from the $S^3 \times M^4$ part of the target space are suppressed.) Two facts directly support this proposal. First, there should be a one-to-one correspondence between vertex operators of the boundary CFT inserted at specific boundary points and vertex operators of the worldsheet theory. Second, according to (3.10), the worldsheet $SL(2,C)$ currents generate the standard $SL(2,C)$ action on the boundary coordinates $x, \bar{x}$.

We obtain further insight in the structure of the correlation functions (4.1) by studying the worldsheets that contribute to it in the semi-classical approximation. The general solution to the equations of motion of (3.1) in the absence of sources is

$$\begin{aligned}
\phi &= \log\left(1+b(z)\bar{b}(\bar{z})\right) + \rho(z) + \bar{\rho}(\bar{z})\,,\\
\gamma &= a(z) + e^{-2\rho(z)}\bar{b}(\bar{z})\left(1+b(z)\bar{b}(\bar{z})\right)^{-1}\,,\\
\bar{\gamma} &= \bar{a}(\bar{z}) + e^{-2\bar{\rho}(\bar{z})}b(z)\left(1+b(z)\bar{b}(\bar{z})\right)^{-1}\,,
\end{aligned} \tag{4.2}$$

for arbitrary holomorphic functions a, b, ρ. The case with sources can be dealt with by allowing poles in a, b, ρ. To find these functions in the presence of arbitrary vertex operators is rather complicated (it is the analogue of the uniformization problem in Liouville theory [19]). We will therefore only consider the behavior of the worldsheet near a single vertex operator

$$V = \left((\gamma - x)(\bar{\gamma}-\bar{x})e^{\phi} + e^{-\phi}\right)^{2j}(z_0) \tag{4.3}$$

at the point $z = z_0$. The relevant equations of motion read

$$\frac{1}{2\pi}\partial\bar{\partial}\phi - \frac{1}{2\pi}e^{2\phi}\partial\bar{\gamma}\bar{\partial}\gamma + 2j\frac{(\gamma-x)(\bar{\gamma}-\bar{x})e^{\phi}-e^{-\phi}}{(\gamma-x)(\bar{\gamma}-\bar{x})e^{\phi}+e^{-\phi}}\delta^{(2)}(z-z_0) = 0\,, \tag{4.4}$$

$$\frac{1}{4\pi}\partial(e^{2\phi}\bar{\partial}\gamma) + 2j\frac{(\gamma-x)e^{\phi}}{(\gamma-x)(\bar{\gamma}-\bar{x})e^{\phi}+e^{-\phi}}\delta^{(2)}(z-z_0) = 0\,, \tag{4.5}$$

$$\frac{1}{4\pi}\bar{\partial}(e^{2\phi}\partial\bar{\gamma}) + 2j\frac{(\bar{\gamma}-\bar{x})e^{\phi}}{(\gamma-x)(\bar{\gamma}-\bar{x})e^{\phi}+e^{-\phi}}\delta^{(2)}(z-z_0) = 0\,. \tag{4.6}$$

[5] From now on we will work with the original variables as they appear in (3.1). Furthermore, we will suppress the Q_5 dependence until the discussion of the central charge after equation (7.6).

[6] This proposal is not complete as it stands; see section 7 for a more precise statement.

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This system has the solution

$$
\begin{aligned}
\phi &= 2j \log|z - z_0|^2 + b + c(z - z_0) + \bar{c}(\bar{z} - \bar{z}_0) + \cdots \\
\gamma &= x + a(z - z_0)^{-4j}(\bar{z} - \bar{z}_0)^{1-4j} - 2ac(z - z_0)^{1-4j}(\bar{z} - \bar{z}_0)^{1-4j} + \cdots \\
\bar{\gamma} &= \bar{x} + \bar{a}(z - z_0)^{1-4j}(\bar{z} - \bar{z}_0)^{-4j} - 2\bar{a}\bar{c}(z - z_0)^{1-4j}(\bar{z} - \bar{z}_0)^{1-4j} + \cdots
\end{aligned}
\tag{4.7}
$$

where a, b, c are some arbitrary constants and the dots indicate higher-order regular terms. The corresponding functions in (4.2) are

$$
\begin{aligned}
a(z) &= x\,, \qquad b(z) = ae^{b}(z - z_0)^{1-4j}\,, \\
\rho(z) &= 2j\log(z - z_0) + \frac{b}{2} + c(z - z_0)\,.
\end{aligned}
\tag{4.8}
$$

Since we consider only vertex operators with $j \leq -1/2$, corresponding to boundary conformal weight $h \geq 1/2$, the worldsheet coordinates at z_0 are $(\phi, \gamma, \bar{\gamma})(z_0) = (\infty, x, \bar{x})$. Thus the worldsheet develops an infinite tube that attaches to the point $(x, \bar{x})$ at the boundary of AdS_3. In the field theory limit, the worldsheet degenerates, and we recover the picture of [4], where boundary correlation functions are expressed in terms of Feynman diagrams consisting of bulk-bulk and bulk-boundary propagators. This is further evidence for the identification (4.1). The structure of the worldsheet is illustrated in figure 1.

If we evaluate the semi-classical contribution to the correlation function (4.1), we encounter a divergence arising from the stretching of the worldsheet to the boundary at infinity of AdS_3. To regularize this divergence, we introduce a worldsheet UV cutoff ϵ, and multiply the correlation function by a suitable power of ϵ before taking the limit $\epsilon \to 0$. The appropriate power is easily determined (see [10] for a similar analysis for Liouville theory) and corresponds to a wave function renormalization for each vertex operator V_j,

$$
V_j^{ren} = \epsilon^{8j^2} V_j\,. \tag{4.9}
$$

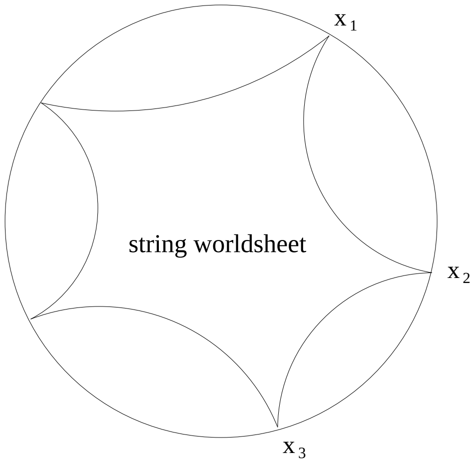

Figure 1: Semi-classical worldsheet in the presence of vertex operators

A similar renormalization has also been found in [16], where correlation functions of V_j with $j > 0$ were studied. In that situation, one consequence of the renormalization was that the $e^{-\phi}$ in V_j could be dropped, leading to an exact free-field representation of the correlation functions. It should be possible to find similar exact free-field representations of the correlation functions of V_j with $j < 0$, because $SL(2, C)$ representations with spins j and $-1-j$ are equivalent. We shall not pursue this further here; nevertheless, we will find that the wave function renormalization brings about many simplifications. In particular, it explains why the free-field approximation is valid, and plays a crucial role in proving the Virasoro Ward identities of the boundary *CFT*.

Besides ϵ, there are two other cutoffs in the problem, the IR cutoff of the bulk theory and the UV cutoff of the boundary *CFT*. All three cutoffs are related. According to (4.7), the bulk IR cutoff U_0 in $U = e^{\phi}$ is

$$U_0 = \epsilon^{4j}\,, \tag{4.10}$$

and depends on which vertex operator is inserted. The UV cutoff $\tilde{\epsilon}$ of the boundary *CFT* is related to U_0 by [11]

$$\tilde{\epsilon} = U_0^{-1}\,. \tag{4.11}$$

With this identification of the cutoff parameters, (4.9) may be expressed in terms of the boundary *CFT* cutoff $\tilde{\epsilon}$ as

$$V_j^{ren} = \tilde{\epsilon}^{2h} V_j\,. \tag{4.12}$$

The factor $\tilde{\epsilon}^{2h}$ matches the scaling behavior of the primary field of the boundary *CFT* corresponding to the worldsheet vertex operator V_j. This fits well with the *AdS*/*CFT* duality.[7] The relation between U_0 and ϵ is illustrated in figure 2.

We next turn to the fluctuations around the semi-classical worldsheet. If we denote the latter by $(\phi_0(z), \gamma_0(z), \bar{\gamma}_0(z))$ and quantum fluctuations by $(\phi_q(z), \gamma_q(z), \bar{\gamma}_q(z))$, we see from (4.7) that the dominant contribution to the kinetic term of the quantum fields near $z = z_0$ is

$$\int d^2z |z - z_0|^{-2} \left(\phi_q(z)^2 + |z - z_0|^{8j} |\gamma_q(z)|^2\right). \tag{4.13}$$

For the action to be finite, we need

$$\phi_q(z_0) \sim \epsilon^{\frac{1}{2}}, \qquad \gamma_q(z_0) \sim \epsilon^{\frac{1}{2} - 4j}. \tag{4.14}$$

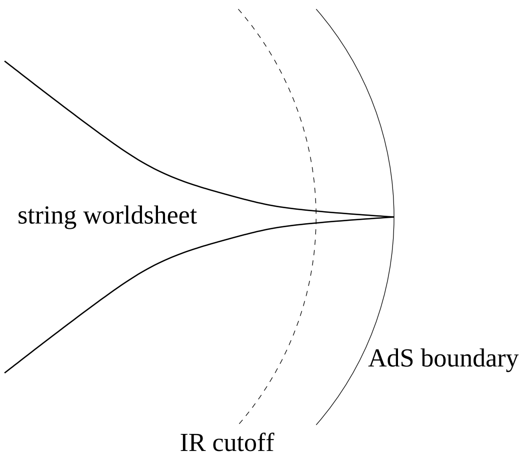

Figure 2: Bulk IR cutoff versus worldsheet UV cutoff

In particular, the fluctuations of the worldsheet vanish near the boundary as we take $\epsilon \to 0$. Furthermore, no quantum terms in the background field expansion of the vertex operators V_j contribute to the correlation function (4.1). Therefore the one-loop worldsheet correction to the correlation function consists only of the determinant of the kinetic term of the quantum fields $(\phi_q(z), \gamma_q(z), \bar{\gamma}_q(z))$.

[7] These relations among the cutoff parameters hold even when we restore the Q_5 dependence.

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5. The Virasoro algebra

So far we have discussed the primary fields of the boundary *CFT*. We now turn our attention to the boundary Virasoro algebra. Let us briefly recall how the Virasoro algebra arises in [1]. First, we define spaces that are asymptotically anti-de Sitter by imposing on the metric the boundary conditions

$$G_{\phi\phi} = 1 + \mathcal{O}(e^{-2\phi})\,, \qquad G_{\phi\gamma} = G_{\phi\bar{\gamma}} = \mathcal{O}(e^{-2\phi})\,,$$
$$G_{\gamma\gamma} = G_{\bar{\gamma}\bar{\gamma}} = \mathcal{O}(1)\,, \qquad G_{\gamma\bar{\gamma}} = \frac{1}{2}e^{2\phi} + \mathcal{O}(1)\,.$$

Next, we consider the group G of diffeomorphisms that preserve these boundary conditions. To each of these one can associate an ADM-type charge that vanishes identically for a subgroup H of diffeomorphisms that decay sufficiently fast at infinity. The algebra of the quotient G/H is the Virasoro algebra. The infinitesimal diffeomorphisms corresponding to the generators L_n are

$$\begin{aligned}
\xi^{\gamma} &= -\gamma^{n+1} + \mathcal{O}(e^{-4\phi})\,,\\
\xi^{\bar{\gamma}} &= \frac{1}{2}n(n+1)\gamma^{n-1}e^{-2\phi} + \mathcal{O}(e^{-4\phi})\,,\\
\xi^{\phi} &= \frac{1}{2}(n+1)\gamma^{n} + \mathcal{O}(e^{-2\phi})\,.
\end{aligned} \tag{5.1}$$

We have given only the holomorphic part of the Virasoro algebra—the full Virasoro algebra consists of the sum of these generators and their complex conjugates. Moreover, our choice of generators is not unique—we could equally well replace γ by $\gamma - \gamma_0$ everywhere.

If we perform one of the infinitesimal diffeomorphisms (5.1) in the worldsheet theory, the result is the insertion of a combined vertex operator for the graviton and the *NS*-*NS* two-form field. This vertex operator is given by

$$L_n = \delta S_n = \int d^2z \left(\frac{1}{2}(n+1)n\gamma^{n-1}(\partial\gamma\bar{\partial}\phi - \bar{\partial}\gamma\partial\phi) + \frac{1}{2}(n+1)n(n-1)\gamma^{n-2}\partial\gamma\bar{\partial}\gamma\right). \tag{5.2}$$

We have neglected subleading terms in (5.1).

Normally, the graviton vertex operator corresponding to a diffeomorphism is on-shell BRST exact and decouples from the theory, as it corresponds to an unphysical graviton. Alternatively, the graviton vertex operator is the sum of a total derivative and equation of motion terms, and the latter can be dropped by the canceled propagator argument [20].

In the case of AdS_3, however, something special happens. Although we can formally write δS_n as $\{Q_{BRST}, X\}$, X is not normalizable, and therefore δS_n is a non-trivial element of the BRST cohomology. Alternatively, as we will show below, the total derivative terms cannot be dropped: in fact, these terms give rise to the contour integral representation of the Virasoro generators of [9]. From either perspective, then, the vertex operators δS_n are physical states of the theory. Since there are no propagating gravitons in three dimensions, they correspond to degrees of freedom living purely on the boundary of AdS_3 (*i.e.*, singletons).

Altogether we are led to identify an insertion of the boundary stress-energy tensor $T(x)$ in a boundary correlation function with the insertion of the vertex operator $T(\phi, \gamma, \bar{\gamma}; x)$ in the worldsheet correlation function given by

$$\begin{aligned} T(x) &= \sum_{n=-2}^{-\infty} L_n x^{-n-2} \\ &= \int d^2 z \left(\frac{1}{(\gamma - x)^3} \left(\partial\gamma \bar{\partial}\phi - \bar{\partial}\gamma \partial\phi \right) - \frac{3}{(\gamma - x)^4} \partial\gamma \bar{\partial}\gamma \right). \end{aligned} \tag{5.3}$$

We saw previously in (3.14) that, for large ϕ, vertex operators behave like bulk-boundary Green's functions, and in particular that they become localized at single points. The same is true for the stress-energy tensor, although this is less obvious from (5.3). Consider for definiteness the second term in (5.3). For large ϕ, this term seems to be subleading compared to the term $e^{2\phi}\partial\bar{\gamma}\bar{\partial}\gamma$. However, we must be careful, because $(\gamma - x)^{-4}$ blows up near $\gamma = x$. Up to terms subleading in $e^{-2\phi}$, the second term in (5.3) can be rewritten as[8]

$$-3 \int d^2 z e^{2\phi} \left(\frac{(\bar{\gamma} - \bar{x})^2}{(\gamma - x)^2} \frac{e^{-2\phi}}{(|\gamma - x|^2 + e^{-2\phi})^2} \right) \partial\gamma \bar{\partial}\gamma \,. \tag{5.4}$$

Since the Brown-Henneaux diffeomorphisms are defined up to subleading terms only, the same is true for T, and we might as well have used (5.4) in our definition of T. For large ϕ, (5.4) behaves as

$$-3 \int d^2 z e^{2\phi} \left(\frac{(\bar{\gamma} - \bar{x})^2}{(\gamma - x)^2} \delta^{(2)}(\gamma - x) \right) \partial\gamma \bar{\partial}\gamma \,. \tag{5.5}$$

This is the analogue of (3.14) for the stress tensor. As in (3.14), it behaves like a bulk-boundary Green's function, and is localized on the boundary of AdS_3.

6. Boundary Ward identity

As a first application of the definition (5.3), we will show that it correctly reproduces the Virasoro Ward identities of the boundary *CFT*. We first discuss the case of a single insertion of the stress-energy tensor and an arbitrary number of primary fields.

[8] For example, one can choose the representative

$$\xi^{\bar{\gamma}} = \frac{\bar{\gamma} - \bar{x}}{(\gamma - x)^2} \frac{e^{-2\phi}}{|\gamma - x|^2 + e^{-2\phi}}$$

in place of (6.1) to find the expression (5.6) for large ϕ.

The case with more than one stress tensor insertion is more complicated and will be discussed later.

Our strategy for proving the Virasoro Ward identities is to perform a change of variables in the path integral corresponding to a Brown-Henneaux diffeomorphism. The diffeomorphism corresponding to $T(x)$ is

$$\begin{aligned} \xi^{\gamma} &= -\frac{1}{\gamma - x} + \mathcal{O}(e^{-4\phi})\,, \\ \xi^{\bar\gamma} &= \frac{1}{(\gamma - x)^3} e^{-2\phi} + \mathcal{O}(e^{-4\phi})\,, \\ \xi^{\phi} &= \frac{-1}{2(\gamma - x)^2} + \mathcal{O}(e^{-2\phi})\,. \end{aligned} \tag{6.1}$$

Let us perform this change of variables on the correlation function

$$\left\langle \prod_i \int d^2 z_i V_{j_i}(z_i, \bar z_i; x_i, \bar x_i) \right\rangle_{\text{worldsheet}}\,. \tag{6.2}$$

There are two contributions: one comes from the variation of the action, yielding $T(\phi, \gamma, \bar\gamma; x)$, while the other comes from the variation of the vertex operators and has the form

$$\begin{aligned} \delta_\xi V_{j_i} = &- \left(\frac{-j_i}{(x - x_i)^2} + \frac{1}{(x - x_i)} \frac{\partial}{\partial x_i} \right) V_{j_i}(x_i) - \\ &- \frac{j_i (\gamma - x_i)^2}{(\gamma - x)^3 (x - x_i)^2} \left(e^{\phi} (\gamma - x_i)(\bar\gamma - \bar x_i) + e^{-\phi} \right)^{2j_i - 1} R\,, \end{aligned} \tag{6.3}$$

where

$$R = e^{-\phi}(\gamma - 3x + 2x_i) + e^{\phi}(\gamma - x_i)(\bar\gamma - \bar x_i)(\gamma - x)\,. \tag{6.4}$$

In the first line of (6.3) we recognize the operator product expansion of $T(x)$ with $V_{j_i}(x_i)$. Using the results (4.7) and (4.14) from the semi-classical analysis, we determine that the remainder, *i.e.*, the second line in (6.3), gives a vanishing contribution to the correlation function. Indeed, the leading term in the background field expansion vanishes, as do all terms containing quantum fields, after taking into account the renormalization factor (4.9). The main reason for this is the explicit factor of $(\gamma - x_i)^2$ in the second line of (6.3).

We have shown that

$$\left\langle T(\phi, \gamma, \bar\gamma; x) \prod_i \int d^2 z_i V_{j_i}(z_i, \bar z_i; x_i, \bar x_i) \right\rangle_{\text{worldsheet}} \tag{6.5}$$

is equal to

$$\sum_i \left(\frac{h_i}{(x - x_i)^2} + \frac{1}{(x - x_i)} \frac{\partial}{\partial x_i} \right) \left\langle \prod_i \int d^2 z_i V_{j_i}(z_i, \bar z_i; x_i, \bar x_i) \right\rangle_{\text{worldsheet}}\,, \tag{6.6}$$

where $h_i = -j_i$. Since both correlation functions have a corresponding meaning in the boundary CFT, this proves the Virasoro Ward identities of the boundary CFT, to all orders in the string worldsheet theory.

This analysis confirms that only the leading large ϕ behavior of the Brown-Henneaux diffeomorphisms is relevant. Had we chosen any other representative, we would still have obtained the correct Virasoro Ward identity. This is because the insertion of a graviton vertex operator corresponding to a diffeomorphism that decays faster, at large ϕ, than the Brown-Henneaux diffeomorphism automatically yields zero. Again, this is as expected.

We can now also make contact with the contour representation of the Virasoro generators in [9]. To do this, we rewrite T in (5.3) as the sum of total derivative and equation of motion terms. The equation of motion terms can be dropped if we view the UV regularization as cutting discs of radius ϵ out of the worldsheet around each of the vertex operators V_{j_i}. The equation of motion terms have only contact-term interactions with the V_{j_i}, and can therefore be neglected. What remains is the total derivative terms. In the presence of the V_{j_i}, the regularized worldsheet acquires a boundary, consisting of the boundaries of the small discs. The total derivative terms thus turn into a sum of contour integrals encircling each of the vertex operators. The relevant contour integrals for $T(x)$ are

$$\sum_i \oint_{z_i} dz \left(\frac{-1}{\gamma - x} e^{2\phi} \partial\bar{\gamma} + \frac{-1}{2(\gamma - x)^2} \partial\phi \right) + \oint_{z_i} d\bar{z} \left(\frac{-1}{(\gamma - x)^3} \bar{\partial}\gamma + \frac{1}{2(\gamma - x)^2} \bar{\partial}\phi \right). \quad (6.7)$$

These contour integrals are just the canonical worldsheet generators of the Brown-Henneaux diffeomorphisms. Therefore, the contour integral can be worked out semi-classically, resulting in in (6.3). All corrections to this semi-classical result vanish as we take the regulator to zero. The contour integral representation of the Virasoro generators in [9] is a slight modification of (6.7), namely,

$$\sum_i \oint_{z_i} dz \left(\frac{-1}{\gamma - x} e^{2\phi} \partial\bar{\gamma} + \frac{-1}{(\gamma - x)^2} \partial\phi + \frac{1}{(\gamma - x)^3} \partial\gamma \right). \quad (6.8)$$

The difference between (6.7) and (6.8) is annihilated when acting on V_{j_i}'s. In the free-field approximation, the integrand of (6.8) contains purely holomorphic operators, and it is valid to use free-field OPE's in computing contour integrals around the V_{j_i}. Again we recover (6.3) up to terms that vanish as the regulator is taken to zero. This shows precisely how and when the free-field representation is exact.

7. $T(x)T(y)$ OPE and central charge

To evaluate the insertion of two or more boundary stress tensors in a correlation function, one might consider, along the lines of the above procedure, performing consecutive Brown-Henneaux diffeomorphisms and studying the resulting Ward identities.

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The only novel feature would be the variation of the stress tensor under a Brown-Henneaux diffeomorphism. As it will turn out, this is not the whole story and has to be supplemented by an additional ingredient. The variation of the stress tensor can be computed using the contour integral representation (6.7). It is easiest to vary a mode of (6.7),

$$\begin{aligned} L_n \equiv & \sum_i \oint_{x_i} dz \left(-\gamma^{n+1} e^{2\phi} \partial\bar\gamma + \frac{1}{2}(n+1)\gamma^n \partial\phi \right) + \\ & + \sum_i \oint_{x_i} d\bar z \left(-\frac{1}{2} n(n+1)\gamma^{n-1} \bar\partial\gamma - \frac{1}{2}(n+1)\gamma^n \bar\partial\phi \right) , \end{aligned} \tag{7.1}$$

under the Brown-Henneaux diffeomorphism (5.1) corresponding to L_m. This yields

$$\begin{aligned} \delta_m L_n = & (m-n) L_{m+n} - (m^3 - m) \sum_i \oint_{z_i} \gamma^{m+n-1} \partial\gamma + \\ & + \frac{1}{2} m(m+1) \sum_i \left(\oint_{z_i} dz \gamma^{n+m} \partial\phi + \oint_{z_i} d\bar z \gamma^{n+m} \bar\partial\phi \right) + \\ & + \frac{1}{4} m(m+1)(n+2m-1) \sum_i \left(\oint_{z_i} dz \gamma^{n+m-1} \partial\gamma + \oint_{z_i} d\bar z \gamma^{n+m-1} \bar\partial\gamma \right) . \end{aligned} \tag{7.2}$$

The two last lines in this expression vanish as we send the regulator to zero. The last term in the first line is similar to the expression for the central charge proposed in [9]. However, since we insert the boundary Virasoro generators at points different from the insertion points of the primary fields, this term does not contribute. The L_n correspond to insertions of T at 0 or ∞, and

$$\oint_{z_i} \gamma^{m+n-1} \partial\gamma = 0 \,, \tag{7.3}$$

if $x_i \neq 0, \infty$.

All that remains from (7.2) is the Virasoro algebra with vanishing central charge. Hence, performing two Brown-Henneaux variations gives us the correct Ward identity for the insertion of two stress tensors in a correlation function of primary fields, except for the central charge term.

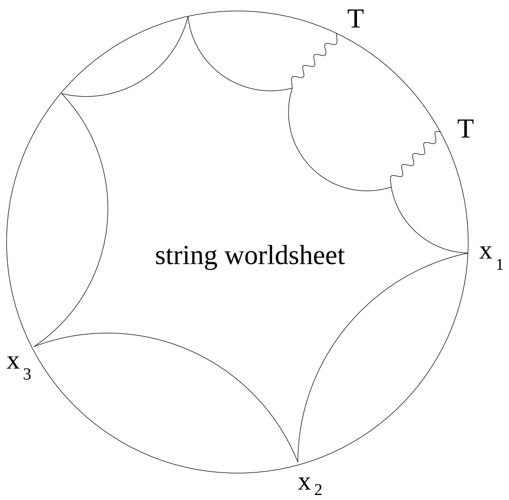

Figure 3: Single string worldsheet contributing to the $\langle TTV_1 \ldots V_n \rangle$ correlator. This diagram does not contribute the central charge of the Virasoro algebra.

The reason that the computation does not capture the central charge in this Ward identity is the following. In the AdS/CFT duality, the string theory on AdS is second-quantized. Therefore we need to sum over all possible string worldsheets, including disconnected ones. This corresponds in the supergravity limit [4] to the prescription to sum over all Feynman diagrams constructed out of bulk-bulk and bulk-boundary propagators, including disconnected Feynman diagrams. So far we have been focusing on a single string worldsheet, as illustrated in figure 3. Let us denote by $\langle\langle V_1 \ldots V_n \rangle\rangle$ the second-quantized string theory correlation function involving arbitrary multiple worldsheets, and by $\langle V_1 \ldots V_n \rangle$ the correlation function obtained from a single worldsheet. Then

$$\langle\langle V_1 \ldots V_n \rangle\rangle = \langle V_1 V_2 \cdots V_n \rangle + \langle V_1 V_2 \rangle \langle V_3 \cdots V_n \rangle + \cdots \tag{7.4}$$

It is $\langle\langle V_1 \ldots V_n \rangle\rangle$, rather than $\langle V_1 \ldots V_n \rangle$, that should be identified with a boundary CFT correlation function. One can easily check that the Virasoro Ward identities still hold if we replace $\langle V_1 \ldots V_n \rangle$ by $\langle\langle V_1 \ldots V_n \rangle\rangle$. However, the correlation function $\langle\langle TTV_1 \ldots V_n \rangle\rangle$ containing two boundary stress-energy tensors includes a contribution from

$$\langle TT \rangle \langle\langle V_1 \ldots V_n \rangle\rangle\,, \tag{7.5}$$

as illustrated in figure 4. We have not yet computed the two-point function of stress tensors. The previous analysis of the Ward identities does not apply to $\langle TT \rangle$, because the contour integral representation of T cannot be used in the absence of other vertex operators.

When $Q_5 \gg 1$, the two-point function of the energy-momentum tensor is computable in the semi-classical approximation giving

$$\langle T(x)T(y) \rangle_{\rm worldsheet} = \frac{c/2}{(x-y)^4}\,, \tag{7.6}$$

with $c = 6Q_1Q_5$. Let us outline the derivation of this formula. As shown in section 5, the energy-momentum tensor $T(\phi, \gamma, \bar\gamma; x)$ can be interpreted as the bulk-boundary Green's function for a graviton in AdS_3. Therefore, in the semi-classical approximation, $\langle T(x)T(y) \rangle_{\rm worldsheet}$ can be identified with the two-point graviton amplitude in the AdS_3 supergravity. The relevant part of the supergravity action is (up to numerical coefficients)

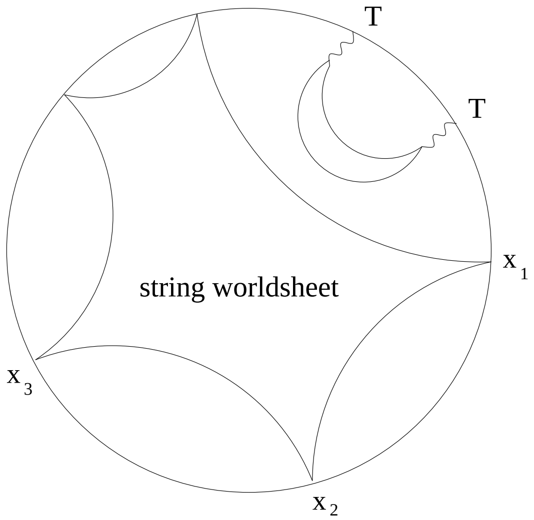

Figure 4: A multiple string worldsheet contributing to the $\langle\langle TTV_1 \ldots V_n \rangle\rangle$ correlator. The central charge $c = 6Q_1Q_5$ is obtained from this diagram.

$$S = \frac{1}{l_p} \int d\phi d\gamma d\bar\gamma \sqrt{g} (R + l_{AdS}^{-2}) + (\text{boundary term})\,. \tag{7.7}$$

If we perturb the metric by $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$, the action is expanded as

$$S = \frac{1}{l_p} \int d\phi d\gamma d\bar\gamma \sqrt{g} \partial h \partial h + \cdots \tag{7.8}$$

Let us choose h to be the bulk-boundary Green's function with sources at x and y on the boundary. Since $\sqrt{g} \sim l_{AdS}^3$ and $\partial^2 \sim l_{AdS}^{-2}$, the action scales as $S \propto l_{AdS}/l_p$. The x, y dependence of the action is determined by the $SL(2, C)$ invariance, and we obtain[9]

$$S \sim \frac{l_{AdS}/l_p}{(x-y)^4} \sim \frac{Q_1 Q_5}{(x-y)^4}\,. \tag{7.9}$$

Thus the Virasoro central charge indeed arises from the two-point graviton amplitude, which is a part of the disconnected diagram in figure 4.

It should also be possible to obtain (7.6) directly from a string worldsheet computation. In string theory, every genus-zero worldsheet carries an extra factor of g_s^{-2}. Therefore the disconnected diagram of figure 4 has an extra factor of $g_s^{-2} = Q_1\sqrt{Q_5}$ compared to the connected diagrams of figure 3. The worldsheet amplitude itself is a function of $l_{AdS}/l_s = \sqrt{Q_5}$ only. For the two-point function of the energy-momentum tensors, our preliminary computation (analogous to the spacetime computation in [22]) indicates that the only l_{AdS} dependence comes from the measure of the ϕ zero mode integral. Thus we expect that this computation also reproduces (7.6) with $c \sim Q_1 Q_5$. It would be desirable to make this computation more precise in order to estimate finite Q_5 corrections to the central charge formula.

8. Discussion

In this paper we have studied string theory on AdS_3 and found that many properties of the AdS/CFT duality can be understood from a semi-classical analysis. In particular, we found vertex operators in the worldsheet theory that correspond to the insertion of operators in the boundary *CFT*. The structure of these vertex operators is somewhat reminiscent of the master field for large N field theory. We showed that the string worldsheet stretches to the boundary of AdS_3 in the presence of such vertex operators, and that the Virasoro generators of Brown and Henneaux directly give rise to the contour integral representation of the Virasoro algebra in [9]. We have explained why, in this representation, the contour is localized near the boundary of AdS_3, and deduced from this the Virasoro Ward identities of the boundary theory. This clarifies several aspects of [9]. However, in our formulation the central charge arises by a different mechanism than one put forth in [9]. We found no need to introduce fundamental strings at infinity and to consider worldsheets wrapping a certain number of times around the boundary of *AdS*. Instead the central charge arose from the disconnected diagram of the second-quantized string theory. It is conceivable that the two different pictures of the central charge are roughly analogous to the short and long string pictures that one encounters, for instance, in matrix string theory [23, 24, 25]. The precise meaning and definition of such a long string picture would require further clarification.

[9] An explicit computation of this can be found in [21].

Several other issues deserve further investigation. We have not yet given a detailed derivation of the central charge from the worldsheet theory. It would be interesting to do this and to see whether the central charge satisfies a non-renormalization theorem in the case of superstrings on $AdS_3 \times S^3 \times M^4$. In addition, we would like to extend this analysis to Lorentzian signature AdS_3, and to have a more detailed understanding of the spectrum and the vertex operators in that case. Finally, we would like to see whether this formulation of string theory on AdS_3 can be used in a practical way to compute higher order α' corrections to supergravity results.

Note Added: Toward the completion of this paper, we received [26]. In that paper, string theory on $AdS_3 \times S^3 \times T^4$ is studied using the approach of [9], and a disagreement is found in the spectra of the $U(1)^4$ charges between the string theory on AdS_3 and the CFT_2 with target space $(T^4)^N/\mathcal{S}_N$. Since that computation depends crucially on the evaluation of the $U(1)$ central charge, it would be interesting to calculate the central charge from our point of view and see if the disagreement persists.

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